The Conspicuous Mountains and Depressions on the Lunar Profile, as Observed in the Beaded Eclipse

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Abstract

Almost all of the past data on the lunar jagged profile have been derived from direct measurements of photographs of the Moon's bright or silhouetted limb. In this paper, data, which were derived from contact times of the moon's limb with the sun's limb, observed during an annular eclipse (or called Beaded Eclipse), will be presented, with special emphasis on the larger mountains and valleys on the lunar profile.

The comparison with Hayn's data shows that the observed mountains are higher, the depressions are deeper, and specially *some* of the valleys are *much* deeper than those in Hayn's data. Further, it seems that our observational results of the depth of large and sharp depressions naturally came out deeper than those of ordinary photographic observations of the Moon's limb.

1. *Introduction.*

On May 9th 1948, a solar eclipse was observed on Rebun Island, Hokkaido, Japan. The magnitude of the eclipse was 0.9996 (Sun's diameter $=1$) and since the eclipsed Sun looked like a bright beaded ring at the middle phase, the eclipse like this should be specially called a "beaded eclipse", as suggested by Prof. Joe Ueta.

The observation corps of the Kyoto University made the observations by the cinematographic method in the central eclipse zone on Rebun Island. Photographs of this beaded eclipse are shown on Plates 1 and 2, enlarged from the cinematographic film; Plate 1 shows a beaded figure at the middle phase, and Plate 2 is a series of six photographs of the beaded eclipse at 2 second intervals.

The photographic equipment was composed of a reflecting tele-photo objective "Fujinami Type-3" and a movie camera, and mounted equatorially (see Plate 3). We operated the instrument for a full minute, containing the beaded phase, and took the cinematograph of the sun and time signals together. The details of the instruments and photographic data are shown in the Appendix I of this paper.

As a first step in the reduction of this cinematographic observation, the time of the middle phase of the eclipse are measured precisely. The results were read before the Spring Meeting (May 1950) of the Astronomical Society of Japan, and the contents are given in the Appendix II. The main results were as follows:

Middle time of the eclipse, T_0:

$$T_0 = 2^h 50^m 32^s.02 \pm 0^s.20, \text{ U.T., May 9th, 1948,} \tag{1.1}$$

Angular distance from Moon's center to Sun's center at the middle phase, a_0:

$$a_0 = +0''.41 \pm 0''.10\,, \tag{1.2}$$

where the positive sign means that the Sun comes to the north side of the Moon.

Difference between Sun's apparent radius (r_s) and that of the Moon (r_m):

$$r_s - r_m = +1''.05 \pm 0''.06\,. \tag{1.3}$$

And, position of observation:

$$\begin{aligned} &\text{geodetic longitude:} && \text{E. } 141^\circ\ 3'\ 44''.810\,, \\ &\text{,,} \qquad \text{latitude:} && \text{N. } 45^\circ\ 22'\ 12''.033\,, \\ &\text{,,} \qquad \text{height:} && 26.02 \text{ meters above sea level.} \end{aligned} \tag{1.4}$$

In the course of the reduction mentioned above, the data of Hayn's table (Karten Von Hayn, Selenographische Koordinaten, 3 Teil) were used for the Moon's jagged profile. But, from the photographic images of this beaded phase, I easily recognized that the Moon's profile, as derived from Hayn's table, is considerably different from the real profile in some portions. In the process of determination of the values (1.1), (1.2) and (1.3) for the middle phase, therefore, only the data of four mountains and fifteen valleys which seem not to show large differences were used.

And, after the reduction was finished, I tried to measure the height and depth of the larger mountains and depressions of the Lunar profile, starting from premised values as (1.1), (1.2) and (1.3).

In what follows, the process of the reduction will be given, as well as the final results and the comparison with Hayn's data.

2. *Method of reduction.*

Figure 1 shows schematically a state near the middle phase of this eclipse, where the dotted circle means the Sun's limb[1], the jagged circle shows exaggeratedly the lunar profile, K is the Moon's center, and S the Sun's center. It is assumed that the mean level of the lunar jagged profile coincides with Hayn's mean level and is a circle with its center at K.

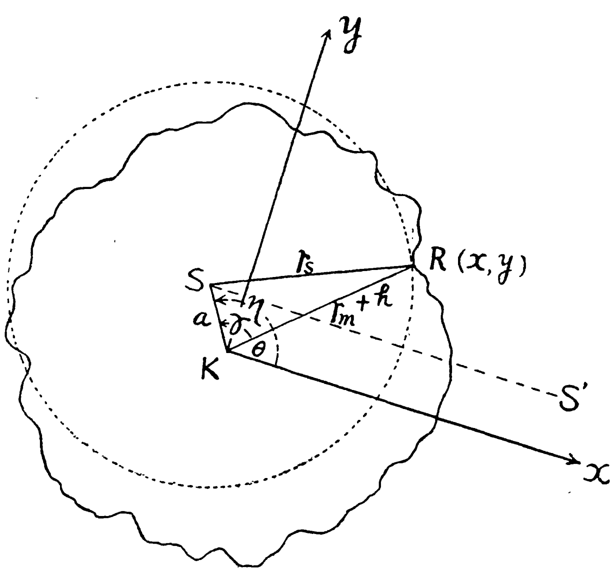

Fig. 1.

A rectangular coordinate system(xy)is set in a plane which is perpendicular to the line of sight, passing the Moon's center, K, the origin being at K. The x-axis is taken parallel to the direction of the Sun's motion ($S \to S'$) relative to the Moon, and the positive y-axis is directed at right angle to the x-axis in

1) Actually, the Sun's limb also may be jagged. But in this paper the solar profile was treated as a circle, because the order of magnitude of the solar jags seems to be smaller than the order, discussed in this paper, and because the photographic exposure in our cinematographic observation, as described in Appendix I, was too short to show the solar chromosphere.

counter-clockwise way. Position angle for a point on the Moon's limb is measured counter-clockwise from the x-axis.

Now, let a point R on the lunar profile be in contact with the Sun's limb, and the height of the point R above the Moon's mean level be h, then the coordinates of the point, x and y, are

$$\begin{aligned} x &= (r_m+h)\cos\theta\,, \\ y &= (r_m+h)\sin\theta\,, \end{aligned} \tag{2.1}$$

where, $\theta=$ position angle of the contact point, and $r_m=$ Moon's mean radius.

In the above equations (2.1), x, y, r_m and h are all given in angular measure, since the difference of tangent and arc of a small angle like the Moon's semi-diameter is negligible. The coordinates of Sun's center, x_s and y_s, at contact are expressed as

$$\begin{aligned} x_s &= +c(T-T_0)\,, \\ y_s &= a_0\,, \end{aligned} \tag{2.2}$$

where, $T=$ the contact time, $T_0=$time of the middle phase of the eclipse, $c=$angular velocity of the Sun's apparent motion relative to the Moon, and $a_0=$angular distance from the Moon's center to the Sun's center at the middle phase of the eclipse, i.e. the shortest distance between the centers of the Moon and the Sun, and its positive value refers to the case where the Sun lies to the north of the Moon's center.

At contact, we have

$$r_s^2 = a^2+(r_m+h)^2-2a(r_m+h)\cos\gamma\,, \tag{2.3}$$

where, $a=$ angular distance between the Moon's and the Sun's centers, $\gamma = \angle RKS = \eta-\theta$, $\eta=$position angle of the Sun's center, measured counter-clockwise from the x-axis.

From the equation (2.3), it comes out

$$r_m+h = a\cos\gamma + \sqrt{a^2\cos^2\gamma-a^2+r_s^2}\,, \tag{2.4}$$

and expanding the square root in series, we have

$$r_m+h = a\cos\gamma + r_s\left(1-\frac{1}{2}\frac{a^2}{r_s^2}\sin^2\gamma-\frac{1}{8}\frac{a^4}{r_s^4}\sin^4\gamma-\cdots\right).$$

In this equation, we can neglect all terms of orders higher than the square of a/r_s. For instance, even for the case as $\theta = 90°$ and $a = 22''$, the effect of the neglected terms is smaller than $0''.001$ for h.

Then, equation (2.4) becomes

$$h = (r_s-r_m)+a\cos\gamma-\frac{1}{2}\frac{a^2}{r_s}\sin^2\gamma\,. \tag{2.5}$$

The height of point R above the Moon's limb is determined by this equation: a and γ in (2.5) are calculated by the following equations:

$$a = \sqrt{a_0^2+c^2(T-T_0)^2}\,, \tag{2.6}$$

$$\gamma = \eta-\theta \tag{2.7}$$

$$\mathrm{tg}\,\eta = \frac{a_0}{c(T-T_0)}\,; \tag{2.8}$$

and values of $(r_s - r_m)$, a_0 and T_0 are shown in the foregoing Paragraph **1**, as the premised values in this reduction.

3. *Data used in the reduction.*

The time signals were printed on the sound track of the cinematographic film on which all figures of the beaded phase were photographed. It enabled us to read the contact times of points on the lunar profile from negative images of the cinematographic film. The contact times were judged from the times of vanishing or appearing of the bright beads, or the instantsw hen the Sun's arc breaks or recovers. The contact times, read out of the film, are shown in Table 1.

In the Table, column " π " gives the position angle of the contact point on the Moon's limb referred to the center, and this angle is measured counter-clockwise from the north direction of the Moon's rotation axis. In the column "Figure", notation **V** means depression and **M** means mountain on the lunar profile. The photographic image of the valley makes a bright bead and the mountain top is recognized as a gap between the beads. In the column " Phenomenon ", notation **d** means contact state by disappearance of the image and **e** means contact state of emergence. The column " Contact time " gives the minutes and seconds of the contact time, after 2^h (U.T.) May 9th, 1948.

Accuracy of the contact times in Table 1 varies with the position angles of the contact points. The greatest accuracy is attained at the position angles corresponding to the direction of the Snn's motion relative to the Moon, i.e. at $\pi = 73°$ and at $73° + 180° = 253°$. The thrice estimations give the mean error of $\pm$ 2 frames of the movie film, corresponding to $\pm 0^s.08$ in time. The accuracy falls down with increasing angular distance in both sides of the place of greatest accuracy. But for the data given in Table 1, the errors do not exceed $\pm$ 5 frames, i.e. $\pm 0^s.2$.

Some of the beads on the southern and northern limbs of the Moon never vanished throughout the cinematographic exposure. Such beads correspond to deep valleys. For such objects, four in number, being located near the north and south poles on the lunar profile, their depths were determined by the following methods and they are indicated with the prefices (※ or ※※) in Table 1.

※ $\pi = 174°$, Valley: the time of the emergence of the bead at the bottom of this valley was estimated to be $49^m54^s.2$, judging from the sequence of the images of beads, by the comparison of this bead and that at $\pi = 203°$, for which the contact time is known already.

※ $\pi = 327°$, Valley: emersion time of this bead was estimated to be $49^m56^s.9$ by the comparison with that at $\pi = 203°$.

※※ $\pi = 164°$, Valley: contact time of this valley could not be estimated by the same method as was used in the above two cases, because the bead for this valley was very bright during the cinematographic exposure. Therefore, by direct comparison of the photographic images in the middle phase, the depth of this valley was estimated to be the same as the valley at $\pi = 174°$, i.e. $h = -1.''45$.

Table 1. The observed data.

No.	π	Figure	Phenomenon	Observed contact time
1	10°	M	d	50^m 37^s.55
2	27	V	,,	38, 20
3	33	M	,,	36, 72
4	39	V	,,	38, 56
5	46	M	,,	35.69
6	59	V	,,	39, 21
7	85	V	,,	33, 85
8	89	M	,,	31.15
9	112	V	,,	49.52
10	125	M	,,	28, 32
11	142	V	,,	59.93
12	154	M	,,	36.22
※※ 13	164	V	—	———
※ 14	174	V	e	49 54, 2
15	190	M	,,	50 40.79
16	203	V	,,	28.16
17	212	M	,,	31.39
18	232	V	,,	27.29
19	243	M	,,	31.80
20	248	V	,,	27.91
21	254	M	,,	31.60
22	268	V	,,	26.61
23	278	M	,,	27.38
24	287	V	,,	25.27
25	314	M	,,	28.52
※ 26	327	V	,,	49 56.9
27	341	M	,,	50 36.68
※※ 28	347	V	—	———
29	354	M	e	50 51.14
30	359	V	,,	55.21

For ※ and ※※, see Paragraph 3.

※※ $\pi = 347°$, Valley: by the same method as above, alike to the valley at $\pi = 359°$, i.e. $h = -0''.25$.

4. *Results of the reduction.*

The heights of several of the larger mountains and depressions on the lunar profile, derived by the method explained in Paragraph **2** from the observed data given in Paragraph **3**, are shown in Table 3. Several reference data in the process of the reduction are shown in Table 2. The current numbers of data are assigned to respective peaks and valleys.

In Table 2, λ and p are selenographic longitude and north polar distance respectively, and x_s, a and γ have the same meanings as in Paragraph 2. In Table 3, values in column h are the heights of the points on the lunar profile above the mean level, and "O" means the values derived from observation, and "C" those taken from Hayn's Table. Column $O-C$ shows difference between the values from our observation and those given in Hayn's Table. P and D are arguments for Hayn's Table.

The Moon's librations at the time of the observation were $\lambda_0 = -5°.40$ and $\beta_0 = -0°.91$, in the selenographic longitude and latitude.

The data from Hayn's Table are given to the order of $0''.1$ merely, but our data are given to the order of $0''.01$. Our data contain some errors of the order of $0''.01$, but they are reliable to the order $0''.1$ at least.

About the results of our observation, we shall first study the amount of the errors of the contact times. The errors were estimated as $\pm\, 0^s.08$ at the Moon's equator and $\pm\, 0^s.2$ at the Moon's poles,

Table 2. Main reference data in calculation process.

No.	x_s	γ		a	λ		p
	″	°	′	″	°	′	°
1	+ 2.02	−100	51	2.06	271	11.3	10
2	+ 2.26	−119	1	2.30	266	56.5	27
3	+ 1.72	−121	54	1.77	266	27.7	33
4	+ 2.39	−131	35	2.43	266	7.8	39
5	+ 1.34	−131	20	1.40	265	50.3	46
6	+ 2.63	−152	27	2.66	265	26.9	59
7	+ 0.67	−155	50	0.78	264	56.7	85
8	− 0.32	− 63	28	0.52	264	52.9	89
9	+ 6.40	−210	38	6.42	264	31.5	112
10	− 1.35	− 64	8	1.41	264	17.5	125
11	+10.21	−242	0.	10.22	263	52.7	142
12	+ 1.54	−241	22	1.59	263	21.0	154
※※ 13	——	——		——	262	25.5	164
※ 14	−13.86	− 98	0	13.86	258	35.0	174
15	+ 3.21	−285	1	3.24	88	10.5	170
16	− 1.41	−141	29	1.47	86	2.6	157
17	− 0.23	−194	57	0.47	85	32.4	148
18	− 1.73	−167	37	1.78	84	58.4	128
19	− 0.08	−244	12	0.42	84	45.6	117
20	− 1.50	−185	33	1.56	84	40.8	112
21	− 0.15	−245	45	0.44	84	35.0	106
22	− 1.98	+158	0	2.02	84	21.9	92
23	− 1.70	+146	8	1.75	84	12.7	82
24	− 2,47	+141	17	2.50	84	3.0	73
25	− 1.28	+105	57	1.34	83	27.2	46
※ 26	−12.85	+108	52	12.86	82	44.0	33
27	+ 1.70	− 69	47	1.75	81	11.0	19
※※ 28	——	——		——	79	32.9	13
29	+ 6.70	− 92	57	7.01	73	39.2	6
30	+ 8.48	− 98	32	8.49	35	17.5	1

For ※ and ※※, see Paragraph 3.

as described in the foregoing paragraph **3**. But in the process of the reduction of the height by the equation (2.5): $h = (r_s - r_m) + a\cos\gamma - \dfrac{a^2 \sin^2\gamma}{2 r_s}$, the error in h becomes very small in all cases, because $\cos\gamma$ approaches to maximum value 1 at the equator and zero at the poles, on the contrary to the magnitude of uncertainty of a which is calculated by $a = \sqrt{a_0^2 + x_s^2}$ and $x_s = c(T - T_0)$. And the effect of third term for this error is smaller than $0''.0001$. From estimated error of the contact time, $\pm 0^s.08$ in T, the error of the depth of $\pi = 248°$ Valley near the Moon's equator, for instance, is $\pm 0''.03$, and for $\pi = 10°$ Mountain near the north pole the error of the height becomes $\pm 0.''01$ from $\pm 0^s.2$ in T.

Table **3**. Height of the mountains and valleys on the lunar profile, derived from our observation, and comparisons with the Hayn's data.

No.	π	P	D	h o	h c	o−c	Figure	
				$''$	$''$	$''$		
1	10°	10°.00	+0°.21	$+0.6_6$	+0.1	$+0.5_6$	**M**	
2	27	26.96	−1.39	-0.0_7	−0.5	-0.4_3	**V**	
3	33	32.95	1.93	$+0.1_2$	−0.1	$+0.2_2$	**M**	
4	39	38.94	2.38	-0.5_6	−0.3	-0.2_6	**V**	
5	46	45.94	2.99	$+0.1_2$	−0.1	$+0.2_2$	**M**	
6	59	58.91	3.90	-1.3_1	−0.6	-0.7_1	**V**	
7	85	84.98	5.03	$+0.3_3$	+0.5	-0.1_7	**V**	
8	89	88.99	5.11	$+1.2_8$	+0.6	$+0.6_8$	**M**	d'Alembert
9	112	112.09	5.07	-4.4_8	−2.0	-2.4_8	**V**	d'Alembert
10	125	125.14	4.66	$+1.6_7$	+1.4	$+0.2_7$	**M**	Rook
11	142	142.16	3.76	-3.7_9	-1.4_5	-2.3_4	**V**	Rook
12	154	154.15	2.91	$+0.2_9$	$+0.5_3$	-0.2_4	**M**	Dörfel
※※ 13	164	164.13	2.08	−1.4	−0.2	−1.2	**V**	
※ 14	174	174.12	1.19	-1.4_5	−1.0	-0.4_5	**V**	Dörfel
15	190	189.99	0.31	$+1.8_8$	+1.1	$+0.7_8$	**M**	Leibniz
16	203	202.95	+1.54	-0.1_0	−0.1	0.0_0	**V**	
17	212	211.92	2.36	$+0.5_9$	+0.3	$+0.2_9$	**M**	
18	232	231.89	3.95	-0.6_9	−0.5	-0.1_9	**V**	
19	243	243.02	4.67	$+0.8_6$	+0.5	$+0.3_6$	**M**	
20	248	247.91	4.93	-0.5_0	−0.1	-0.4_0	**V**	
21	254	253.93	5.20	$+0.8_7$	+0.7	$+0.1_7$	**M**	Smyth
22	268	267.99	5.63	-0.8_2	−1.5	$+0.6_8$	**V**	Smyth
23	278	278.04	5.73	-0.4_0	-0.0_7	-0.4_7	**M**	
24	287	287.08	5.69	-0.9_0	-0.7_7	-0.1_3	**V**	
25	314	314.17	4.70	$+0.6_8$	+0.7	-0.0_2	**M**	
※ 26	327	327.21	3.95	-3.1_9	−1.9	-1.2_9	**V**	Humbolt
27	341	341.21	2.86	$+1.6_5$	+0.3	$+1.3_5$	**M**	
※※ 28	347	347.21	2.34	−0.2	+0.1	−0.3	**V**	
29	354	354.24	1.68	$+0.6_6$	+0.3	$+0.3_3$	**M**	
30	359	359.42	0.82	-0.2_5	-0.4_2	$+0.1_7$	**V**	

For ※ and ※※, see Paragraph **3**.
For errors of data in column o (observational results), see Paragraph **4**.

Consequently, the observational errors which are most relevant to the height should be found in the value of $r_s - r_m$, which is one of the premised values in the reduction, i.e. $r_s - r_m = +1.''06 \pm 0''06$. The probable error $\pm 0''.06$ in the value of $r_s - r_m$ directly affect the height of the lunar profile.

The base value of $r_s - r_m$: $+1''.05$ utterly modified the mean level of the Moon. The heights on the lunar profile, calculated by the equation (2.5), are affected directly by the value of $r_s - r_m$. For example, if a certain smaller value than $+1''.05$ be assigned for $r_s - r_m$, the heights of all points on the lunar profile will become lower, and also if some larger value than $+1''.05$ is applied the reverse case will happen; but the difference in height between the mountain top and valley bottom never differs from that in the case of the premised value. Whereas, according to our results some mountain peaks are higher and some valleys are deeper than Hayn's values. It seems that the value $r_s - r_m = +1''.06$ is reasonable and reliable.

The following table summarizes the comparison between data, based on our observation and those of Hayn's Table. We find some of the depressions are especially deep.

$O-C$	Total	Place (No. in Table 3)
+ for mountains; higher than Hayn's data.	11	1, 3, 5, 8, 10, 15, 17, 19, 21, 27, 29
− for monntains; lower than Hayn's data.	3	12, 23, 25
− for depressions; deeper than Hayn's data.	12	4, 6, 7, 9, 11, 13, 14, 18, 20, 24, 26, 28
+ for depressions; shallower than Hayn's data.	3	2, 22, 30
just coincide	1	16

If the observational errors are less than $0''.1$, it can be said that our observation and Hayn's Table coincide only at three places, No. 16, 24 and 25 in Table 3, and if the observational errors are as large as $0''.2$, we are not able to improve Hayn's data, so far, of ten places, No. 3, 5, 7, 12, 16, 18, 21, 24, 25 and 30. But, in spite of the scrutiny, twenty places show large discrepancies.

Depressions and mountains which have large values of $O-C$ are,

Depression d'Alembert;	$O-C = -2''.48$,
,, Rook;	,, $= -2.34$,
Mare Humbolt,	,, $= -1.29$,
,, Smythii,	,, $= +0.68$,

and at the position angle $\pi = 164°$, corresponding to a part of the Depression Dörfel, there is a very deep valley which is not given in Hayn's Table, where $O-C$ is $-1''.2$.

Mountain d'Alembert,	$O-C = +0''.68$,
,, $\pi = 10°$,	,, $= +0.56$.

The beaded eelipse is a better opportunity to observe precisely the extreme heights of mountains and valleys on the lunar profile compared with the case of the bright Moon. Under ordinary circumstances, the extreme bottoms of the sharp valleys and small jags are not measurable from the photograph of the bright Moon, according to the effect of the irradiation. At the time of an eclipe, the valleys will show fully their depths on the background of the Sun. So, it seems quite natural that our observational results of the depth of large and sharp depressions are deeper than those derived from ordinary photographic observations of the Moon's bright limb.

From the appearance of our cinematographic images, it is perceived that the crescent of the Sun's limb breaks into many beads at the time of contact, and the total number of these small beads is above two hundreds, and therefore it is recognized that Hayn's Table gives only rough estimates for the many small jags on the lunar profile. But in this paper, only larger mountains and depressions were selected for study, and very small beads were excluded.

I am very glad to express my thanks here to Professor Joe Ueta for his kind discussions and advices.

Appendix I. Photographic and instrumental data on the cinematographic observations of the Beaded Eclipse on May 9th 1948.

We made the observation at the northern point of this annular eclipse zone of Rebun Island, to determine the central line of the eclipse, in co-operation with the southern corps. Our program was to take the cinematograph of the partial and annular phases of the eclipse together with time signals, with a photographic telescope. The work was performed successfully under a very clear weather. The photographs, enlarged from our movie film, at the middle phase, are shown in Plates 1 and 2.

The place of observation was Common Fishing Ground No. 21, Kitousu, Kabuka-Mura, Rebun Island, Hokkaido, Japan, Lougitude: E 141° 3′ 44″.810, Latitude: N 45° 22′ 12″.033, Height: 26.02 meters above sea level.

The arrangements of the instruments at the ground of observation are shown in Plate 3, and the instrumental and photographic data are as follows,

Mounting: Equatorial mounting with driving clock.

Photographic telescope: Reflecting tele-photo objectives FUJINAMI Type-3 (Cassegrainian type), focal length 1400 millimetre, aperture ratio f/15.

Camera: Parvo type-*K*.

Film: FUJI Sound Recording Negative Film, resolving power 60/mm.

Filter: Sunglass, with the exposure coefficfent about 100 times for the white light.

Sensitive color region by using the film in conjunction with the filter: 420—500 $\mu\mu$.

Exposure: For the first and last contacts, *f*/18, 1/432 second, and for the beaded phases, *f*/15, 1/50 second.

Recording mechanism for the Time Signals: A lamp box with a tiny electric discharging bulb is attached compactly on the upper side of the aperture plate of the Parvo camera. On a side of the lamp box there is a small hole, and it is so arranged that the sound-track of the cine-film passed very close by the small hole and the distance between the respective centers of the hole and the exposed frame is just two frames. The discharging bulb is lighted by current through a relay when the primary circuit of the chronometer is closed, so that, on the sound-track of the developed film black dots are printed consecutively for every half second of the chronometer. And it is so adjusted that the last one of the consecutive dots gives just the beginning of each second of the chronometer. The time error of the time mark is estimated to be below $\pm 0^s.02$ when the film-speed is 24 frames per second. By these time marks the time of exposure of every frame is determined relative to the chronometer. The chronometer was corrected by the wireless-time-signal, sent by the Eclipse Time Service.

The members of our observation corps were as follows; Shigetsugu FUJINAMI, leader; Saburo ISAYAMA, Masao KASHIMA, Tomoichi KURAMOCHI, cameramen; Toshikazu HIGAMI, Shigejiro NISHIMURA, Eikichi KAN, assistant astronomers; Minoru KONDO, Shigeyuki SEKINE, Yoshio YAMAGUCHI, assistant cameramen.

Our observation corps has got the aid from the SHOCHIKU Movie Studio, Ltd. Co.. The camera men were sent with almost all of the photographic materials from the Studio in Kyoto. I express my hearty thanks for the kind encouragement of the directors and members of the company, and Goro TAKAHASHI, director of the Kyoto Studio.

Appendix II. The difference of radii and the relative position of the Sun and Moon, derived from the cinematographic observation of the beaded eclipse on May 9th 1948.

Firstly by the reductian of the movie film of the beaded eclipse, photographed with the photographic telescope and movie camera which are described in Appenpix I, the difference of radii and the relative position of the Sun and Moon were determined. In this Appendix, I will report the results as well as the process of the reduction. The method of the reduction is analogous to that K. Kordylewski used for a total eclipse.[1]

At a contact state of the Sun and the Moon, as shown in Fig. 1, a relation between the Sun's center $S(x_s, y_s)$ and the contact point $R(x, y)$ on the Moon's limb is given in the following form:

$$r^2_s = (y-a_0)^2+(x-x_s)^2 . \tag{6.1}$$

By (2.2) it becomes

$$x-c(T-T_0) = \pm\sqrt{r^2_s-(y-a_0)^2} .$$

Put

$$\pm\sqrt{r^2_s-(y-a_0)^2} = A , \tag{6.2}$$

then we have

$$T = T_0+\frac{1}{c}(x-A) . \tag{6.3}$$

T in the above equation is the contact time, derived from the prediction of the eclipse. Accordingly this T will be denoted by T_c, to be distinguished from the observed contact time. Then

$$T_c = T_0+\frac{1}{c}(x-A) . \tag{6.4}$$

Here A has the $\pm$ sign, as seen from (6.2). But the sign may be suitably selected, because the calculated contact time is not much different from the observed contact time. Using the equation (6.4) together with (2.1), T_c may be calculated from the values of r_m, θ and h.

Now, we consider $T=f(T_0, x, y, a_0, r_s)$, then we have a differential relation between those quantities from (6.3) as follows,

$$cdT = cdT_0+dx-\frac{r_s}{A}dr_s+\frac{y-a_0}{A}dy-\frac{y-a_0}{A}da_0 \tag{6.5}$$

Here we put

$$\left.\begin{aligned} r_s \sin\delta &= (y-a_0) \\ r_s\cos\delta &= A \end{aligned}\right. \tag{6.6}$$

where, δ is the position angle of the contact point, referred to the Sun's center, measured counter-clockwise from the direction of the Sun's motion relative to the Moon. ($\delta = \angle S'SR$, in Fig. 1)

From (2.1),

$$\left.\begin{aligned} dx &= \cos\theta \cdot dr_m \\ dy &= \sin\theta\cdot dr_m \end{aligned}\right. \tag{6.7}$$

Replacing $y-a_0$, A, dx and dy in (6.5) by the relations of (6.6) and (6.7),

$$cdT = cdT_0-\frac{1}{\cos\delta}dr_s-\frac{\sin\delta}{\cos\delta}da_0+\frac{\cos(\theta-\delta)}{\cos\delta}dr_m .$$

It will be allowed to put $\cos(\theta-\delta)\simeq 1$, because $\theta\simeq\delta$. Then, the equation becomes

$$cdT = cdT_0-\frac{\sin\delta}{\cos\delta}da_0-\frac{1}{\cos\delta}d(r_s-r_m) . \tag{6.8}$$

We introduce the following notations for simplicity

$$\left.\begin{aligned} X &= dT , \\ Y &= \frac{1}{c}da_0 \\ Z &= \frac{1}{c}d(r_s-r_m) , \\ T_{obs}-T_c &= dT , \end{aligned}\right. \tag{6.9}$$

where, T_{obs} is the observed contact time. Then, from (6.8) we have

$$\cos\delta\cdot X-\sin\delta\cdot Y-Z = (T_{obs}-T_c)\cos\delta \tag{6.10}$$

This equation is the observation-equation. The three unknowns, Y, Y and Z, in the equation will be determined by the method of the least squares.

Then we have

$$T_0 = \underline{T_0}+X$$
$$a_0 = \underline{a_0}+cY \qquad (6.11)$$
$$(r_s-r_m) = \underline{(r_s-r_m)}+cZ$$

where, the under-lined terms $\underline{T_0}$, $\underline{a_0}$ and $\underline{(r_s-r_m)}$ are the assumed values at the start of the raduction, and T_0, a_0 and (r_s-n_m) are the resulting values.

The data for the reducton are shown in Table 4, where T_{obs}: observed contact time, T_c: calculated contact time, using the Hayn's data of the lunar profile, h: height of the contact point from the Moon's mean level derived from Hayn's Table, and other symbols have the same meanings as in Table 1.

The assumed values at the start of this reduction, derived from the preliminary calculation, are as follows,

$$\underline{T_0} = 50^{m}32^{s}.09, \quad \underline{a_0} = +0''.50, \quad \underline{r_s-r_m} = +1''.00\,.$$

The other fundamental data were derived from the American Ephemeris and Nautical Almanac 1948, for the station as $c = 0''.366$ and $r_s = 951''.00$.

Nineteen data as shown in Table 4 have been selected out of the observed contact times of many beads, such that they do not differ too much from the calculated times with Hayn's data. Here, I excluded those, having the residuals exceed $\pm 3^{s}$, because it seems that their heights, derived from Hayn's Table, deviate too much from the real ones. For example, on the deepest point at $\pi = 112°$ in the Depression d'Alembert, the observed contaet time is $50^{m}49^{s}.52$, while the calculated contact time is $41^{s}.09$, assuming Hayn's datum: $h = -2''.0$, and the residual becomes $+8^{s}.4$. On another deeper point at $\pi = 107°$ in the same depression, the observed contact time: $50^{m}47^{s}.90$, the calculated contact time: $37^{s}.75$ (Hayn's datum: $h = -1''.6$), and the residual becomes $+10^{s}.1$. Also, another deep potnt $\pi = 104°$ in the same depression, $T_{obs} = 50^{m}45^{s}.06$, while $T_c = 37^{s}.54$ (Hayn's $h = -1''.0$), and the residual $= +7^{s}.5$. For those points where the residuals reach to such large amount, Hayn's data might differ greatly from the real profile of the Moon. The data of those points were not used in this reduction.

Using the nineteen observation-equations in Table 4, by the method of least-squares, the normal equations are derived as follows:

$$12.388\,X+1.296\,Y+\ 0.161\,Z = -1.098\,,$$
$$1.296\,X+6.168\,Y+\ 1.768Z = -1.338\,,$$
$$0.161\,X+1.761\,Y+19.000\,Z = +2.234\,.$$

From those, the unknowns and the probable errors are,

$$X = -0^{s}.065 \pm 0^{s}.207, \quad Y = -0^{s}.244 \pm 0^{s}.297, \quad Z = +0^{s}.141 \pm 0^{s}.168\,.$$

Accoordingly, $X = dT_0 = -0^{s}07 \pm 0^{s}.21$, $cY = da_0 = -0''.09 \pm 0''.11$, $cZ = d(r_s-r_m) = +0''.05 \pm 0''.06$, and then, by the equation (6.11) we have

$$T_0 = 2^{h}50^{m}32^{s}.02 \pm 0^{s}.21\,, \quad \text{(U.T.)}$$
$$a_0 = +0''.41 \pm 0''.11\,,$$
$$r_s-r_m = +1''.05 \pm 0''.06\,.$$

It may be said, from the value of a, our observation corps was located at a distance 850 ± 240 meters from the central line of the eclipse (The running direction of the Moon's shadow-center, calculated by the American Ephemeris, bears an azimuth E 32° N at Rebun Island).

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§ *Postscript.*

Editors of this publication noticed me of H. Kristenson's paper on the lunar profile. It will be good here to look over results of him and others.

Kristenson's paper[2] shows a new method to determine the contacts of the solar eclipse by the spectrophotometric method, which was applied to the total eclipse on July 9th 1545. There, also he gives features of the lunar periphery, derived from direct measurements of partial phases, besides that derived by the spectrophotometric method.

The spectrophotometric lunar profile in Kristenson's paper was derived from the flash spectra, extending a little over some 50° of periphery respectively at the second and third contacts of the

Table 4. Data in the reduction to determine the difference of radii and the relative position of the Sun and Moon.

No.	Figure	Phenomenon	π	T_{obs}	h	x	$y-a_0$	A	T_c	$T_{obs}-T_c$	Observation equation, $\cos\delta\cdot X-\sin\delta\cdot Y-Z=(T_{obs}-T_c)\cos\delta$	Residual
			°	s	s	″	″	″	s	s	s	s
R 1	V	e	287	25.27	−0.77	+827.80	+464.07	+830.08	25.85	−0.58	+0.873X −0.488Y − z = −0.506	+0.49
〃 2	〃	〃	280	26.88	−0.1	+878.85	+359.94	+880.25	28.26	−1.38	+ .926 〃 − .378 〃 − 〃 = −1.278	+1.26
〃 3	M	〃	278	27.38	+0.07	+885.11	+344.59	+886.37	28.64	−0.76	+ .932 〃 − .362 〃 − 〃 = −0.708	+1.17
〃 4	V	〃	313	28.52	+0.7	+541.21	+781.11	+542.47	28.56	−0.04	+ .570 〃 − .821 〃 − 〃 = −0.023	+0.07
〃 5	〃	〃	310	28.85	+0.6	+581.32	+751.64	+582.61	28.56	−0.29	+ .613 〃 − .790 〃 − 〃 = +0.178	−0.29
〃 6	〃	〃	226	29.13	−0.3	+808.01	−499.54	+809.24	31.48	−2.35	+ .535 〃 + .525 〃 − 〃 = −1.257	−0.35
〃 7	〃	〃	222	29.34	−0.1	+771.40	−544.80	+772.40	29.37	−0.03	+ .812 〃 + .583 〃 − 〃 = −0.024	+0.04
〃 8	〃	〃	216	30.78	+0.2	+709.45	−632.60	+710.08	30.36	+0.42	+ .747 〃 + .665 〃 − 〃 = +0.314	−0.33
〃 9	M	d	89	31.15	+0.6	−932.17	−186.77	−932.48	32.94	−1.79	− .981 〃 + .196 〃 − 〃 = +1.756	−1.77
〃 10	〃	e	212	31.39	+0.3	+663.70	−680.62	+664.20	30.74	+0.65	+ .698 〃 + .716 〃 − 〃 = +0.454	−0.46
〃 11	〃	〃	215	32.42	+0.3	+698.79	−644.95	+698.89	31.74	+0.68	+ .735 〃 + .678 〃 − 〃 = +0.500	−1.25
〃 12	V	d	84	33.04	+0.5	−944.76	−104.80	−945.21	33.32	−0.28	− .994 〃 + .110 〃 − 〃 = +0.278	−0.28
〃 13	〃	〃	85	33.85	+0.5	−942.79	−121.27	−943.24	33.31	+0.54	− .992 〃 + .128 〃 − 〃 = −0.536	+0.53
〃 14	〃	〃	87	35.32	+0.5	−938.01	−154.10	−938.43	33.25	+2.07	− .987 〃 + .162 〃 − 〃 = −2.043	+2.02
〃 15	〃	〃	92	35.73	+0.2	−920.76	−235.20	−921.46	33.99	+1.74	− .967 〃 + .247 〃 − 〃 = −1.683	+1.68
〃 16	〃	〃	34	37.14	−0.1	−675.19	+667.65	−677.23	37.67	−0.53	− .712 〃 − .702 〃 − 〃 = +0.377	−0.51
〃 17	〃	〃	21	37.23	−0.1	−521.52	+793.43	−524.28	39.63	−2.40	− .551 〃 − .834 〃 − 〃 = +1.322	−1.33
〃 18	〃	〃	27	38.20	−0.5	−601.40	+734.26	−604.37	40.21	−2.01	− .636 〃 − .772 〃 − 〃 = +1.278	−1.29
〃 19	〃	〃	39	38.56	−0.3	−741.17	+593.29	−743.24	37.75	+0.81	− .782 〃 − .624 〃 − 〃 = −0.633	+0.61

eclipse. In comparison between the photometric profile and Hayn's, as the mean deviation for a single point of the profile Kristenson gives a value $0''.21$. According to Kristenson's description, Hayn himself has estimated the mean error of his profile to be about $0''.25$ and it means that the photometric results from the flash spectra agree pretty well with Hayn's in the observed region of the lunar profile. The slit used in the photometric observation has a dimension of $0''.35$ on the lunar periphery, it is expected to give results a little smoothed, but our observation is concerned entirely with the extreme of mountain top and valley bottom on the lunar periphery.

The other results of lunar profile in Kristenson's paper, which was derived from the direct measurements of large scale photographs taken during the partial phase, shows large deviations from Hayn's profile, in Kristenson's of the profile. And similar tendency is perceived, referred to the deviations from Hayn's profile, in Kristenson's results and in our case, such that the jagged feature of the lunar profile seems to be much exaggerated compared with Hayn's. Another example of such tendency will be found in K. Graff's lunar profile[3], derived from the photograph taken during the annular eclipse 1912 April 17, which shows the large discrepancies from Hayn's at some points of the profile.

These large differences of height at some points on the lunar profile, derived from the direct measurements of photographs, may be caused by accidental refraction in the terrestrial atmosphere as Kristenson stated, and also by irradiation effects in photographic emulsion. But, the whole amount of these differences in height may not be explicable by accidental effects merely. In our observational results, the irradiation effect is not expected since our results were derived from the contact times which were judged from the instants of vanishing or appearing of the beads, or the instants when the Sun's arc breaks or reunites, as stated on Paragraph **3**.

After all, librations in selenographic longitude and latitude in the case of our observation differ so much ($10°$ in longitude, $0°.7$ in latitude) from those in the case of Kristenson that both observational results are not able to be compared directly. So it is regrettable that the validity of our result can not be assured by Kristenson's results.

References

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